MATHEMATICS SPECIALIST

MAWA Semester 2 (Unit 1 & Unit 2) Examination 2017

Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **the end of week 6 of term 4, 2017**

Section One: Calculator-free (50 Marks)

Question 1(a)

Solution

$\underset{\sim}{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$\Rightarrow \underset{\sim}{a} - 3\underset{\sim}{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ -23 \end{pmatrix}$

Marking key/mathematical behaviours	Marks
• calculates and uses vectors in component form	1

Question 1(b)

Solution

$|\underset{\sim}{a}| = \sqrt{3^2 + (-2)^2} = \sqrt{13}$

Marking key/mathematical behaviours	Marks
• calculates the magnitude of a vector	1

Question 1(c)

Solution

$\hat{\underset{\sim}{a}} = \frac{1}{\sqrt{13}}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Marking key/mathematical behaviours	Marks
• Uses the magnitude of a vector	1
• Uses the scalar multiple of a vector	1

Question 1(d)

Solution

$\underset{\sim}{c} = |\underset{\sim}{a}| \times \hat{\underset{\sim}{b}} \qquad = \sqrt{13} \times \frac{1}{5\sqrt{2}}\begin{pmatrix} 1 \\ 7 \end{pmatrix}$

$= \frac{\sqrt{13}}{5\sqrt{2}} \times \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

Marking key/mathematical behaviours	Marks
• uses the magnitude vector $\underset{\sim}{a}$	1
• represents $\underset{\sim}{c}$ a scalar multiple of vector $\underset{\sim}{b}$	1

Question 2

Solution	
Position vectors of A, B and C = $\begin{pmatrix}3\\-1\end{pmatrix},\begin{pmatrix}-1\\15\end{pmatrix}$ and $\begin{pmatrix}9\\-25\end{pmatrix}$, $\Rightarrow AB=\begin{pmatrix}-4\\16\end{pmatrix}, AC=\begin{pmatrix}6\\-24\end{pmatrix}$ $\begin{pmatrix}-4\\16\end{pmatrix}=\lambda\begin{pmatrix}6\\-24\end{pmatrix}$ $\Leftrightarrow AB//AC \left(\text{where } \lambda=\frac{-2}{3}\right)$ $\Leftrightarrow ABC$ is a straight line	
Marking key/mathematical behaviours	Marks
• calculates column vector AB • calculates column vector AC • represents AB as a scalar multiple of a vector AC • examines scale factors of the vectors and determines they are parallel	1 1 1 1

Question 3(a)

Solution	
$2x^2+6x+5=0$ $x=\frac{-6\pm\sqrt{6^2-4(2)(5)}}{4}$ $x=\frac{-6\pm 2i}{4}$ $x=\frac{-3\pm i}{2}$	
Marking key/mathematical behaviours	Marks
• Determines the two complex roots	2

Question 3(b)

Solution

$$\frac{3+i}{7i-1}\times\frac{7i+1}{7i+1}=\frac{-4+22i}{-49-1}$$
$$=\frac{-4+22i}{-50}$$
$$=\frac{-2+11i}{-25}$$

Marking key/mathematical behaviours	Marks
• uses the correct term to rationalise the quotient	1
• determines the simplified result	1

Question 4

Solution

$$RTP:\ \overline{\left(\frac{z}{w}\right)}=\frac{\overline{z}}{\overline{w}}\Leftrightarrow\overline{\left(\frac{a+bi}{c+di}\right)}=\left(\frac{a-bi}{c-di}\right)$$

$$\overline{\left(\frac{a+bi}{c+di}\right)}=\overline{\left(\frac{a+bi}{c+di}\times\frac{c-di}{c-di}\right)}$$
$$=\overline{\left(\frac{ac+bd+(bc-ad)i}{c^2+d^2}\right)}$$
$$=\frac{ac+bd-(bc-ad)i}{c^2+d^2}$$
$$=\frac{ac-bci+bd+adi}{c^2+d^2}$$
$$=\frac{c\ \ a-ib\ \ +id\ \ a-bi}{c+di\ \ \ c-di}$$
$$=\frac{a-bi}{c-di}\times\frac{c+di}{c+di}$$
$$=\left(\frac{a-bi}{c-di}\right)$$

Marking key/mathematical behaviours	Marks
• sets up the expression $\overline{\left(\frac{z}{w}\right)}$ for left hand side in terms of a, b, c & d	1
• modifies the expression by rationalizing the denominator	1
• calculates the conjugate term	1
• factorises the previous expression to obtain the expression $\frac{\overline{z}}{\overline{w}}$	1

Question 5(a)

Solution

The number of ways to arrange the couple and teenage daughter = 3!

Treating the couple and daughter as 1 unit, the number of ways to arrange the 4 persons/unit = 4!

Hence, the number of ways of the 6 people forming a queue

$= 3! \times 4!$
= 6 x 24
= 144

Marking key/mathematical behaviours	Marks
• indicates that the number of ways to arrange the couple and teenage daughter = 3! and treating the couple and daughter as 1 unit, number of ways to arrange 4 persons/unit = 4!	1
• determines the correct value	1

Question 5(b)

Solution

$$r - 8 + r = 20$$
$$r = 14$$

Marking key/mathematical behaviours	Marks
• determines that $r - 8 + r = 20$ for equation to be true and determines the correct value for r	1

Question 5(c)

Solution

$$\frac{m!}{\left(m-(m-4)\right)!} = 210$$
$$\frac{m!}{4!} = 210$$
$$m! = 210 \times 4!$$
$$m! = 7 \times 6 \times 5 \times 4!$$
$$m! = 7! \qquad \therefore m = 7$$

Marking key/mathematical behaviours	Marks
• states the correct equation for ${}^{m}P_{m-4}$	1
• determines the correct value for m	1

Question 5(d)

Solution

Number of arrangements such that all letters are different = ${}^{8}P_{4} = {}^{8}C_{4} \times 4! = 1680$

Number of arrangements such that there is 1 pair of repeated letters =
${}^{2}C_{1} \times {}^{6}C_{2} \times {}^{4}P_{2} = 2 \times 15 \times 12 = 360$

Number of arrangements with 2 pairs of repeated letters = $\dfrac{4!}{2!2!} = 6$

Total number of arrangements = 1680+360+6=2046

Marking key/mathematical behaviours	Marks
• Determines the correct number of arrangements for all different letters	1
• Determines the correct number of arrangements with 1 pair of repeated letters	1
• Determines the correct number of arrangements with 2 pairs of repeated letters	1
• Determines the correct answer	1

Question 6(a)

Solution

$$\text{LHS} = \frac{1}{\tan x} - \frac{1}{\tan 2x}$$
$$= \frac{\cos x}{\sin x} - \frac{\cos 2x}{\sin 2x}$$
$$= \frac{\cos x}{\sin x} - \frac{\left(2\cos^2 x - 1\right)}{2\sin x \cos x}$$
$$= \frac{\cos x . 2\cos x - \left(2\cos^2 x - 1\right)}{2\sin x \cos x}$$
$$= \frac{2\cos^2 x - 2\cos^2 x + 1}{2\sin x \cos x}$$
$$= \frac{1}{2\sin x \cos x}$$
$$= \frac{1}{\sin 2x}$$
= cosec 2x

Marking key/mathematical behaviours	Marks
• expresses tan x and cot x in terms of sin x and cos x	1
• expresses sin 2x and cos 2x in terms of sin x and cos x	1
• simplifies and equates to the RHS correctly	1

Question 6(b)

Solution

$$\begin{aligned}\sqrt{2}\cos x+\sqrt{2}\sin x &= R\cos(x-\alpha)\\ &= R(\cos x\cos\alpha+\sin x\sin\alpha)\\ &= 2(\cos x\cos\alpha+\sin x\sin\alpha) \text{ where } R=2\\ &= 2\left(\cos x\frac{1}{\sqrt{2}}+\sin x\frac{1}{\sqrt{2}}\right) \text{ where } \cos\alpha=\sin\alpha=\frac{1}{\sqrt{2}}\\ &= 2\cos\left(x-\frac{\pi}{4}\right)\end{aligned}$$

Marking key/mathematical behaviours	Marks
• determines the correct value for R	1
• determines the correct value for α	1

Question 7

Solution

$\overrightarrow{OB}=\overrightarrow{OY}+\overrightarrow{YB}$

$\mathbf{b}=h(\mathbf{b}+\frac{1}{2}(\mathbf{a}-\mathbf{b}))-k(-\mathbf{b}+\frac{1}{2}\mathbf{a})$

$b=\frac{1}{2}h\mathbf{b}+\frac{1}{2}h\mathbf{a}+k\mathbf{b}-\frac{1}{2}k\mathbf{a}$

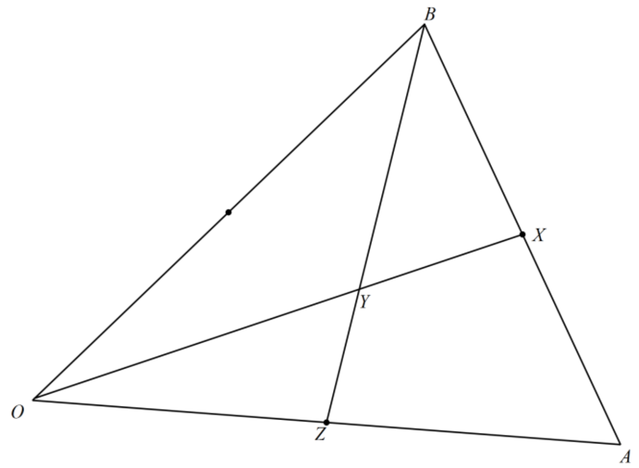
$(1-\frac{1}{2}h-k)\mathbf{b}=\frac{1}{2}(h-k)\mathbf{a}$

Since $\mathbf{a}$ is not parallel to $\mathbf{b}$ then

$(h-k)=0$ and $(1-\frac{1}{2}h-k)=0$

$\Leftrightarrow 1-\frac{3}{2}h=0$

$\Leftrightarrow h=\frac{2}{3}=k$



Marking key/mathematical behaviours	Marks
• Sets up an expression for OY	1
• Sets up an expression for YB	1
• Uses scalars h and k	1
• Determines an equation in terms of $\mathbf{a},\mathbf{b}$, h and k	1
• Solves for h and k	1

 See next page

Question 8(a)

Solution	
Given $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \Rightarrow \text{Det } A = 2 - 12 = -10$ $A^{-1} = -\frac{1}{10}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$	
Marking key/mathematical behaviours	Marks
• evaluates the determinant • modifies the matrix elements correctly	1 1

Question 8(b)

Solution	
$AX = \begin{bmatrix} 13 \\ 2 \end{bmatrix}$ $\Leftrightarrow X = A^{-1}\begin{bmatrix} 13 \\ 2 \end{bmatrix}$ $\Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{10}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 2 \end{bmatrix}$ $\Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$	
Marking key/mathematical behaviours	Marks
• expresses as matrix equation • pre-multiplies by inverse • obtains a matrix expression for unknowns • simplifies answer	1 1 1 1

Question 9(a)

Solution	
$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -1 & 0 \end{bmatrix}$	
Marking key/mathematical behaviours	Marks
• determines reflection matrix • determines shear matrix • multiplies in correct order	1 1 1

Question 9(b)

Solution	
Determinant = -1 Hence the area is unchanged = $\frac{1}{2}(7)(5)=17.5$	
Marking key/mathematical behaviours	Marks
• states determinant = -1 • determines original area	1 1

Question 9(c)

Solution	
$\begin{bmatrix}-2 & -1\\-1 & 0\end{bmatrix}^{-1}=\frac{1}{-1}\begin{bmatrix}0 & 1\\1 & -2\end{bmatrix}=\begin{bmatrix}0 & -1\\-1 & 2\end{bmatrix}$	
Marking key/mathematical behaviours	Marks
• calculates the determinant • determines the inverse	1 1